English Life Tables No. 16 (2000-2002)

Methodology

This document supports the report on ELT16 in Population Trends No 136 and gives a description of the methodology used to construct the English Life Tables (2000-2002).

1. Calculation of the `exposed to risk' and the central rates of mortality, England and Wales, 2000-2002

English life Tables No. 16, 2000-02

Estimates of the `central exposed to risk of death' have been derived from the ONS estimates of the home population (that is, the persons usually resident in England and Wales, excluding foreign visitors and UK Armed Forces stationed elsewhere, but including civilian residents temporarily elsewhere, and all Armed Forces stationed in England and Wales) in mid-2000, mid-2001 and mid-2002. However, these estimates do not give estimates of the number of men and women at individual ages over 90. Neither do population censuses provide reliable estimates of peoples' ages over 90, due largely to the difficulty of obtaining accurate information from very elderly people (or those caring for them) as to their date of birth. Subsequent death registrations, however, are believed to be generally more accurate in reporting age at death. Thus, accurate mortality rates can only be calculated for advanced ages when a population has become extinct. Population numbers can then be obtained by accumulating registered deaths backwards in time, assuming that ages at death have been accurately registered and that international migration is negligible at these ages.

The survivor ratio method (Thatcher, Kannisto and Andreev, 2002) is a modified version of the extinct generations method that does not involve waiting until all the members of a cohort are dead. At old ages only a small proportion of the original member of a cohort will still be alive. It is assumed that the ratio of the number of survivors to the number in the cohort who died over a given period can be estimated from the experience of previous cohorts. This ‘survivor ratio’ can then be applied to the number of deaths for a cohort over the same number of years to obtain estimates for the number of survivors.

The youngest age ω at which there are no survivors for year T is estimated. The numbers of people born in earlier cohorts (before year T-ω) can then be calculated by adding back deaths for those cohorts in each preceding year. The survivor ratio for age ω-1 is then calculated as the ratio of the sum of the population aged ω-1 in years T-1, T-2, T-5, to the total deaths (at ages ω-2, ω-3, …, ω-6) in the preceding 5 years to give an estimate of the number of survivors to age ω-1 in year T. The numbers for that year of birth in earlier years are then calculated by adding back the deaths in each preceding year for that cohort and the whole process is then repeated. By starting at a year later than 2001 the number at each age 90 and over in 2000, 2001 and 2002 were then estimated.

Having obtained mid-year estimates of the population at individual ages 90 and over to supplement those for younger ages taken directly from the ONS estimates, the ‘central exposed to risk of death’ was then derived for each age as follows.

Let $P_{t,x}$ be the home population of a particular sex t years after 1 January 2000, for a particular age group, represented by x. Then, for the particular age group x let the mid year home population estimates at 30 June 2000, 2001 and 2002 be $P_{½,x}$, $P_{1½,x}$, $P_{2½,x}$, respectively.

For the age group x the central exposed to risk of death E_x is defined as

$E_x = \int_0^3 P_{t,x}\, dt = (P_{½,x} + P_{1½,x} + P_{2½,x})$

on the assumption that $P_{t,x}$ is a linear function of *t* (for each age group) over each of the individual years represented by the *t* values 0-1, 1-2 and 2-3. This methodology is the same as that used for ELT 15.

The numbers of deaths used in constructing the tables are those published by ONS. These include all deaths occurring in England and Wales, including those of residents from abroad whose deaths are registered here. It is assumed that inclusion of deaths of abroad-residents approximately compensates for the exclusion of deaths abroad of residents of England and Wales.

Let $\theta_{k,x}$ represent the deaths of a particular sex in calendar year 2000+k at attained age x, with k = 0, 1 and 2. Then the crude central rates of mortality at that age are given, by

$$\overset{\circ}{m}_x = \frac{\theta_{0,x} + \theta_{1,x} + \theta_{2,x}}{E_x} = \frac{\theta_{0,x} + \theta_{1,x} + \theta_{2,x}}{P_{1/2,x} + P_{1\,1/2,x} + P_{2\,1/2,x}}$$

Finally, the graduated central rates of mortality (m_x) were converted to initial rates *(q_x)* for the purposes of the life tables using the methodology outlined in section 2. The resulting crude central rates of mortality (rounded to give decimal places) are set out in Table l. These crude rates were then graduated to run smoothly from age to age (these graduated central mortality rates are also shown in Tables 1-5).

Table 1 Derivation of central rates of mortality (m_x) for 2000-02, England and Wales

(a) Males

Age	Exposed to risk	Deaths 2000-02	Crude death rates per 100,000	Graduated death rates per 100,000		Age	Exposed to risk	Deaths 2000-02	Crude death rates per 100,000	Graduated death rates per 100,000
0	912861	5507	603			60	761951	8097	1063	1057
1	928792	409	44	45		61	755240	8828	1169	1169
2	949719	253	27	25		62	761823	9795	1286	1287
3	971890	158	16	17		63	751985	10562	1405	1412
4	983120	148	15	15		64	733048	11256	1536	1551
5	992397	119	12	13		65	708980	12182	1718	1711
6	1000208	141	14	12		66	680788	12906	1896	1892
7	1012260	120	12	12		67	657537	13988	2127	2100
8	1034554	109	11	11		68	640659	15012	2343	2339
9	1054397	110	10	11		69	631972	16659	2636	2612
10	1065712	127	12	12		70	617707	17887	2896	2928
11	1062501	141	13	13		71	594854	19446	3269	3286
12	1056545	156	15	14		72	566294	20653	3647	3676
13	1047649	195	19	17		73	541140	21902	4047	4098
14	1037250	226	22	21		74	518470	23708	4573	4554
15	1028462	252	25	27		75	494280	24974	5053	5044
16	1015419	360	35	38		76	465991	25999	5579	5568
17	1002419	531	53	52		77	436616	26949	6172	6126
18	975462	721	74	67		78	414023	28096	6786	6721
19	958859	723	75	76		79	396040	29701	7499	7387
20	970861	767	79	78		80	375929	30809	8195	8138
21	972149	730	75	78		81	317611	28240	8891	8989
22	946198	766	81	79		82	251201	24586	9787	9952
23	921419	717	78	80		83	198640	21525	10836	11046
24	922628	753	82	81		84	175270	21564	12303	12282
25	953012	788	83	82		85	162990	22413	13751	13604
26	991442	783	79	84		86	144647	21818	15084	14996
27	1039904	917	88	86		87	124219	20191	16254	16450
28	1094165	959	88	89		88	101652	18449	18149	17957
29	1147179	1119	98	92		89	80592	16078	19950	19507
30	1170206	1113	95	96		90	66335	13711	20669	21105
31	1189579	1196	101	99		91	50370	11321	22476	22896
32	1200629	1269	106	104		92	37292	9449	25338	24941
33	1225673	1307	107	108		93	26693	7327	27449	27281
34	1240857	1407	113	113		94	18443	5534	30006	29964
35	1252999	1424	114	119		95	12293	4052	32962	33009
36	1249057	1543	124	125		96	7977	2821	35364	36181
37	1237823	1655	134	132		97	5010	1993	39780	39393
38	1214962	1627	134	140		98	3010	1266	42060	42606
39	1188614	1787	150	148		99	1776	771	43412	45885
40	1154299	1905	165	158		100	1023	487	47605	49396
41	1123507	1954	174	170		101	555	273	49189	53157
42	1097757	2015	184	184		102	281	166	59075	57183
43	1074567	2169	202	201		103	142	68	47887	61492
44	1046244	2232	213	221		104	78	42	53846	66101
45	1010213	2475	245	245		105	39	29	74359	71031
46	991765	2674	270	273		106	15	11	73333	76300
47	982636	3021	307	302		107	7	5	71429	81930
48	977880	3170	324	333		108	4	3	75000	87943
49	974282	3686	378	365						
50	983188	3985	405	397						
51	1005966	4265	424	430						
52	1053518	4838	459	463						
53	1113482	5568	500	503						
54	1084570	5871	541	551						
55	1024168	6242	609	608						
56	937644	6553	699	678						
57	909549	6933	762	761						
58	859150	7276	847	853						
59	798308	7546	945	952						

Table 1 Derivation of central rates of mortality (m_x) for 2000-02, England and Wales

(b) Females

Age	Exposed to risk	Deaths 2000-02	Crude death rates per 100,000	Graduated death rates per 100,000		Age	Exposed to risk	Deaths 2000-02	Crude death rates per 100,000	Graduated death rates per 100,000
0	871621	4237	486			60	785750	5120	652	639
1	886750	299	34	31		61	781531	5597	716	702
2	906011	163	18	21		62	789462	6079	770	772
3	927335	151	16	16		63	781118	6643	850	850
4	937106	139	15	13		64	766047	7259	948	937
5	943142	111	12	12		65	746911	7655	1025	1034
6	949743	98	10	11		66	725220	8386	1156	1144
7	961881	93	10	10		67	709395	8994	1268	1270
8	984698	108	11	10		68	703417	9938	1413	1414
9	1004921	93	9	10		69	707612	11106	1570	1579
10	1014700	100	10	10		70	707546	12383	1750	1768
11	1011154	102	10	10		71	698029	13943	1997	1986
12	1005498	105	10	10		72	678143	15122	2230	2237
13	998391	99	10	11		73	661004	16513	2498	2513
14	988271	140	14	13		74	648152	18163	2802	2814
15	974167	142	15	16		75	637582	20143	3159	3142
16	958941	220	23	22		76	623429	21899	3513	3497
17	947127	234	25	27		77	608128	23739	3904	3880
18	927349	277	30	28		78	600750	25805	4295	4298
19	934721	273	29	28		79	600494	28686	4777	4777
20	959520	259	27	29		80	593221	31835	5366	5331
21	965768	284	29	29		81	521784	31024	5946	5973
22	944822	269	28	29		82	431486	28942	6708	6718
23	925583	285	31	30		83	361369	26907	7446	7585
24	928248	268	29	30		84	338296	29181	8626	8553
25	952588	308	32	31		85	335047	32354	9657	9606
26	989786	347	35	32		86	317054	33918	10698	10743
27	1039190	354	34	34		87	289255	34589	11958	11965
28	1096871	374	34	36		88	251661	33737	13406	13299
29	1152588	425	37	39		89	214282	32174	15015	14813
30	1183059	481	41	42		90	181880	29900	16439	16536
31	1202656	586	49	45		91	150529	27718	18414	18501
32	1214047	592	49	49		92	121831	24993	20514	20741
33	1238257	662	53	53		93	95410	22024	23084	23117
34	1253340	758	60	58		94	72259	18027	24948	25507
35	1266318	839	66	63		95	53174	14698	27641	27863
36	1267435	876	69	69		96	38070	11531	30289	30138
37	1258205	893	71	75		97	26698	8686	32534	32491
38	1237072	1030	83	83		98	18200	6503	35731	35039
39	1209566	1060	88	91		99	11948	4549	38073	37800
40	1171963	1135	97	99		100	7555	3153	41734	40790
41	1137812	1211	106	109		101	4580	2028	44279	44031
42	1109520	1326	120	120		102	2651	1294	48812	47545
43	1085723	1514	139	132		103	1465	767	52355	51356
44	1057888	1530	145	146		104	799	441	55194	55490
45	1025962	1645	160	161		105	421	235	55819	59975
46	1009165	1877	186	178		106	210	124	59048	64844
47	1000309	1969	197	196		107	103	71	68932	70131
48	996157	2139	215	215		108	49	35	71429	75873
49	991811	2367	239	237		109	18	19	105556	82112
50	1002246	2653	265	260		110	6	3	50000	88891
51	1025211	2839	277	285		111	3	1	33333	96261
52	1072710	3334	311	312		112	2	2	100000	104276
53	1131351	3723	329	341						
54	1101745	4133	375	373						
55	1041103	4179	401	407						
56	954278	4264	447	445						
57	925952	4533	490	486						
58	876830	4553	519	532						
59	818978	4751	580	582						

Table 2 Derivation of central rates of mortality (m_x) for 2000-02, Scotland
(a) Males

Age	Exposed to risk	Deaths 2000-02	Crude death rates per 100,000	Graduated death rates per 100,000	Age	Exposed to risk	Deaths 2000-02	Crude death rates per 100,000	Graduated death rates per 100,000
0	79959	495	619	619	53	108027	726	672	712
1	82387	25	30	33	54	103512	821	793	795
2	85189	22	26	26	55	96583	827	856	886
3	87999	14	16	21	56	87387	847	969	984
4	89093	28	31	18	57	85907	938	1092	1091
5	90379	21	23	16	58	83648	984	1176	1206
6	91288	11	12	15	59	79040	1044	1321	1329
7	93422	10	11	14	60	76383	1180	1545	1460
8	96299	8	8	14	61	75471	1236	1638	1600
9	98486	18	18	14	62	76000	1381	1817	1749
10	99064	16	16	15	63	74345	1446	1945	1906
11	98665	19	19	16	64	72674	1461	2010	2077
12	98986	17	17	17	65	70511	1570	2227	2268
13	99597	15	15	19	66	68370	1714	2507	2482
14	100480	23	23	23	67	65732	1816	2763	2723
15	99888	32	32	31	68	63761	1859	2916	2994
16	98153	43	44	46	69	61984	2045	3299	3298
17	95837	79	82	68	70	60032	2159	3596	3643
18	93506	96	103	92	71	57493	2306	4011	4030
19	97388	107	110	112	72	54215	2423	4469	4452
20	100494	100	100	124	73	51052	2541	4977	4909
21	100975	120	119	128	74	48363	2681	5543	5402
22	95853	135	141	131	75	45799	2614	5708	5933
23	89345	113	126	134	76	43033	2689	6249	6504
24	86017	120	140	137	77	39094	2752	7039	7116
25	86178	114	132	139	78	36489	2869	7863	7771
26	87834	122	139	142	79	33883	2887	8520	8470
27	90516	138	152	145	80	32194	2943	9141	9213
28	94895	159	168	147	81	26631	2632	9883	10004
29	100566	153	152	150	82	21173	2214	10457	10864
30	104170	166	159	152	83	16313	2076	12726	11810
31	107786	153	142	156	84	14429	1929	13369	12851
32	109953	197	179	160	85	13348	1993	14931	13997
33	113363	171	151	166	86	11803	1875	15886	15261
34	114257	195	171	172	87	10175	1770	17396	16655
35	116321	209	180	179	88	8190	1584	19341	18195
36	116488	202	173	187	89	6453	1292	20022	19896
37	117686	237	201	196	90	5478	1146	20920	21778
38	117278	253	216	207	91	4127	901	21832	23862
39	116341	239	205	219	92	3141	730	23241	26169
40	114926	237	206	233	93	2194	626	28532	28649
41	112614	336	298	248	94	1509	437	28960	31265
42	110724	288	260	266	95	945	320	33862	34012
43	108727	320	294	286	96	617	218	35332	36883
44	106885	324	303	309	97	379	155	40897	39871
45	104211	323	310	334	98	223	99	44395	42966
46	102153	381	373	363	99	145	65	44828	46154
47	100099	415	415	396	100	84	31	36905	49422
48	98330	424	431	433	101	38	27	71053	52755
49	97491	472	484	475	102	20	7	35000	56135
50	97715	512	524	523	103	7	6	85714	59542
51	100023	604	604	578					
52	102771	647	630	640					

Table 2 Derivation of central rates of mortality (m_x) for 2000-02, Scotland (b) Females

Age	Exposed to risk	Deaths 2000-02	Crude death rates per 100,000	Graduated death rates per 100,000	Age	Exposed to risk	Deaths 2000-02	Crude death rates per 100,000	Graduated death rates per 100,000
0	76525	370	484	484	53	109920	432	393	434
1	78517	27	34	32	54	105602	478	453	476
2	80508	20	25	23	55	98942	517	523	522
3	83319	10	12	17	56	90019	476	529	572
4	84446	15	18	14	57	89668	616	687	627
5	85508	9	11	11	58	88380	597	675	687
6	86965	13	15	10	59	85024	631	742	754
7	89290	6	7	9	60	82966	781	941	829
8	92397	7	8	9	61	82555	766	928	911
9	94280	6	6	9	62	82993	827	996	1002
10	94693	11	12	9	63	82081	852	1038	1103
11	93731	8	9	10	64	81146	961	1184	1214
12	93576	11	12	11	65	80098	1096	1368	1339
13	94273	18	19	13	66	78643	1101	1400	1477
14	95357	14	15	16	67	76774	1250	1628	1630
15	95780	19	20	21	68	76146	1341	1761	1801
16	94434	25	26	27	69	75528	1544	2044	1991
17	92633	37	40	33	70	75162	1631	2170	2202
18	91066	37	41	38	71	73227	1739	2375	2438
19	96123	33	34	42	72	70349	1917	2725	2701
20	100112	45	45	43	73	67487	2133	3161	2995
21	100039	38	38	43	74	65514	2227	3399	3323
22	95825	34	35	41	75	63902	2388	3737	3689
23	89303	36	40	41	76	62169	2610	4198	4099
24	87077	39	45	41	77	59004	2705	4584	4557
25	88707	38	43	41	78	57270	2897	5058	5068
26	91832	43	47	43	79	55569	3028	5449	5633
27	95230	48	50	45	80	55154	3490	6328	6258
28	100532	49	49	47	81	48187	3243	6730	6950
29	107443	42	39	50	82	40352	3093	7665	7715
30	111950	63	56	53	83	32816	2903	8846	8560
31	115207	67	58	57	84	31183	2999	9617	9494
32	117954	70	59	61	85	30411	3204	10536	10525
33	120997	85	70	66	86	29053	3315	11410	11662
34	122491	91	74	71	87	26139	3280	12548	12918
35	124036	100	81	78	88	22453	3236	14412	14302
36	125380	98	78	85	89	18781	2937	15638	15828
37	126506	106	84	93	90	16055	2816	17540	17509
38	125393	129	103	103	91	13236	2583	19515	19359
39	123512	149	121	113	92	10757	2313	21502	21397
40	120705	146	121	125	93	8388	1929	22997	23630
41	118777	161	136	138	94	6374	1679	26341	26035
42	116545	189	162	152	95	4664	1375	29481	28614
43	114435	198	173	168	96	3240	1029	31759	31371
44	111269	203	182	185	97	2190	776	35434	34307
45	107640	197	183	204	98	1476	542	36721	37426
46	104797	266	254	225	99	970	401	41340	40726
47	102509	268	261	247	100	613	246	40131	44206
48	100366	278	277	272	101	350	174	49714	47864
49	99467	290	292	299	102	217	102	47005	51641
50	99545	316	317	328	103	114	47	41228	55487
51	101711	381	375	361					
52	104359	425	407	396					

Table 3 Derivation of central rates of mortality (m_x) for 2000-02, Northern Ireland
(a) Males

Age	Exposed to risk	Deaths 2000-02	Crude death rates per 100,000	Graduated death rates per 100,000	Age	Exposed to risk	Deaths 2000-02	Crude death rates per 100,000	Graduated death rates per 100,000
0	33555	192	572	579	53	28984	160	552	557
1	34579	12	35	35	54	28519	182	638	615
2	35570	11	31	26	55	27821	197	708	678
3	36505	10	27	21	56	27060	181	669	749
4	37081	5	13	18	57	26798	209	780	827
5	37328	5	13	15	58	25847	220	851	914
6	37298	2	5	14	59	24098	247	1025	1010
7	37462	7	19	13	60	22439	275	1226	1117
8	38230	7	18	13	61	21623	255	1179	1234
9	39270	11	28	14	62	21340	290	1359	1365
10	39970	6	15	15	63	21120	333	1577	1510
11	40262	8	20	18	64	20532	345	1680	1671
12	40655	10	25	21	65	19829	394	1987	1850
13	41232	6	15	27	66	18915	405	2141	2047
14	41616	12	29	35	67	18108	402	2220	2266
15	41670	22	53	48	68	17588	429	2439	2508
16	41141	22	53	62	69	17103	468	2736	2776
17	40952	31	76	76	70	16576	503	3035	3072
18	38487	41	107	88	71	15793	530	3356	3401
19	36300	35	96	95	72	14931	559	3744	3764
20	35062	40	114	100	73	14306	614	4292	4167
21	34128	41	120	105	74	13669	662	4843	4613
22	33125	40	121	108	75	13099	704	5374	5107
23	31976	33	103	111	76	12140	685	5643	5653
24	32022	32	100	112	77	11162	697	6244	6259
25	32215	39	121	113	78	10288	743	7222	6930
26	32935	35	106	112	79	9531	765	8026	7672
27	34072	44	129	111	80	8956	819	9145	8495
28	35307	32	91	108	81	7905	737	9323	9406
29	36559	30	82	106	82	6629	675	10183	10415
30	37101	37	100	105	83	5439	597	10976	11532
31	37373	41	110	105	84	4481	619	13814	12770
32	37631	38	101	105	85	3899	584	14978	14141
33	37985	35	92	107	86	3352	497	14827	15660
34	38405	52	135	110	87	2910	494	16976	17342
35	38900	38	98	114	88	2306	436	18907	19206
36	39029	61	156	120	89	1784	393	22029	21270
37	38712	45	116	127	90	1422	304	21378	23558
38	37851	54	143	135	91	1065	239	22441	26057
39	37027	60	162	146	92	789	211	26743	28717
40	36093	67	186	159	93	566	151	26678	31535
41	35362	66	187	174	94	390	110	28205	34504
42	34509	66	191	192	95	263	82	31179	37615
43	33567	71	212	211	96	165	74	44848	40857
44	33168	58	175	232	97	90	41	45556	44219
45	32350	79	244	255	98	51	25	49020	47684
46	31850	94	295	281	99	29	18	62069	51233
47	31174	100	321	310	100	16	13	81250	54848
48	30415	111	365	341	101	7	4	57143	58504
49	29668	106	357	376	102	4	5	125000	62178
50	29225	108	370	415	103	3	2	66667	65843
51	29149	138	473	458					
52	28914	153	529	505					

Table 3 Derivation of central rates of mortality (m_x) for 2000-02, Northern Ireland
(b) Females

Age	Exposed to risk	Deaths 2000-02	Crude death rates per 100,000	Graduated death rates per 100,000	Age	Exposed to risk	Deaths 2000-02	Crude death rates per 100,000	Graduated death rates per 100,000
0	31711	151	476	481	53	30190	106	351	352
1	32578	14	43	23	54	29591	111	375	381
2	33556	4	12	18	55	28831	123	427	413
3	34650	2	6	15	56	27885	105	377	450
4	34805	6	17	13	57	27913	160	573	491
5	35126	5	14	11	58	26947	143	531	537
6	35247	6	17	10	59	25326	160	632	590
7	35877	4	11	9	60	23697	156	658	650
8	36716	2	5	9	61	23168	179	773	718
9	37470	4	11	8	62	22984	202	879	794
10	37958	2	5	9	63	22931	202	881	880
11	38302	1	3	9	64	22505	215	955	976
12	38738	4	10	10	65	21997	220	1000	1083
13	39149	6	15	11	66	21300	262	1230	1204
14	39530	3	8	13	67	20870	275	1318	1339
15	39658	11	28	15	68	20712	341	1646	1492
16	39578	7	18	17	69	20678	344	1664	1663
17	39557	9	23	20	70	20503	391	1907	1857
18	37492	7	19	22	71	20169	431	2137	2075
19	35275	6	17	24	72	19547	481	2461	2322
20	33957	15	44	26	73	19104	484	2534	2600
21	33030	9	27	28	74	18818	561	2981	2916
22	32730	13	40	29	75	18193	556	3056	3273
23	32224	7	22	31	76	17505	588	3359	3678
24	32271	7	22	32	77	16635	650	3907	4138
25	32517	11	34	32	78	16149	759	4700	4661
26	33228	7	21	33	79	15477	764	4936	5250
27	34558	17	49	34	80	14632	850	5809	5904
28	36073	11	30	35	81	13185	841	6378	6630
29	37399	18	48	37	82	11465	833	7266	7432
30	37871	16	42	38	83	9891	789	7977	8319
31	38659	16	41	41	84	8961	807	9006	9296
32	38893	16	41	43	85	8416	884	10504	10372
33	39365	18	46	47	86	7751	878	11328	11555
34	39628	18	45	51	87	6972	897	12866	12852
35	40075	20	50	56	88	6015	853	14181	14271
36	40287	28	70	61	89	5007	766	15299	15823
37	40108	26	65	68	90	4079	741	18166	17516
38	39358	27	69	76	91	3307	660	19958	19359
39	38563	33	86	86	92	2640	589	22311	21362
40	37909	44	116	98	93	2051	508	24768	23536
41	36930	54	146	113	94	1562	395	25288	25889
42	36037	54	150	128	95	1148	371	32317	28410
43	35114	51	145	145	96	795	277	34843	31074
44	34136	49	144	162	97	528	187	35417	33877
45	32466	59	182	181	98	344	117	34012	36812
46	31143	68	218	201	99	227	83	36564	39870
47	30403	61	201	221	100	147	79	53741	43042
48	29884	60	201	241	101	92	50	54348	46313
49	29543	74	250	262	102	57	29	50877	49671
50	29379	85	289	283	103	31	22	70968	53098
51	29547	79	267	303					
52	29912	98	328	326					

Table 4 Derivation of central rates of mortality (m_x) for 2000-02, England

(a) Males

Age	Exposed to risk	Deaths 2000-02	Crude death rates per 100,000	Graduated death rates per 100,000	Age	Exposed to risk	Deaths 2000-02	Crude death rates per 100,000	Graduated death rates per 100,000
0	864831	5240	606	603	53	1049017	5225	498	512
1	879062	383	44	45	54	1021960	5508	539	563
2	898629	241	27	25	55	965073	5861	607	620
3	918853	152	17	17	56	882832	6156	697	684
4	928856	139	15	15	57	855583	6536	764	755
5	937407	113	12	13	58	807010	6832	847	834
6	944678	132	14	12	59	749262	7039	939	923
7	955901	117	12	12	60	715006	7573	1059	1022
8	976477	108	11	12	61	709396	8261	1165	1134
9	994731	105	11	12	62	716164	9179	1282	1258
10	1005240	120	12	12	63	707168	9879	1397	1398
11	1001504	139	14	13	64	689198	10490	1522	1555
12	995790	152	15	15	65	666178	11391	1710	1731
13	986986	181	18	17	66	639485	12059	1886	1927
14	977345	210	21	21	67	617600	13066	2116	2144
15	968863	244	25	28	68	602209	14084	2339	2385
16	957055	341	36	38	69	594069	15577	2622	2651
17	945352	493	52	53	70	580565	16699	2876	2945
18	920794	678	74	66	71	558882	18197	3256	3270
19	905162	671	74	75	72	532181	19347	3635	3629
20	914839	727	79	78	73	508403	20506	4033	4025
21	916498	670	73	78	74	486605	22152	4552	4463
22	894589	726	81	79	75	463373	23345	5038	4945
23	874227	671	77	80	76	436590	24250	5554	5476
24	877350	708	81	81	77	409352	25139	6141	6062
25	907257	737	81	82	78	388379	26314	6775	6706
26	944820	741	78	84	79	371774	27840	7488	7415
27	991525	859	87	85	80	353396	28890	8175	8195
28	1042998	904	87	87	81	298418	26508	8883	9052
29	1092985	1053	96	90	82	235883	23082	9785	9994
30	1114385	1061	95	93	83	186311	20145	10813	11028
31	1132475	1143	101	96	84	164902	20182	12239	12162
32	1143154	1199	105	100	85	153610	21100	13736	13406
33	1166809	1214	104	105	86	136412	20593	15096	14770
34	1180743	1328	112	110	87	117188	19048	16254	16263
35	1191205	1330	112	116	88	95933	17385	18122	17876
36	1186249	1461	123	123	89	76062	15150	19918	19608
37	1175046	1565	133	131	90	62630	12909	20612	21462
38	1152482	1545	134	140	91	47556	10728	22559	23442
39	1127238	1672	148	151	92	35209	8948	25414	25550
40	1094231	1791	164	163	93	25202	6916	27442	27788
41	1064902	1847	173	176	94	17414	5226	30010	30159
42	1040080	1908	183	191	95	11606	3828	32983	32663
43	1017559	2043	201	208	96	7532	2668	35422	35299
44	990418	2109	213	227	97	4730	1886	39873	38068
45	955500	2350	246	247	98	2842	1196	42083	40968
46	937302	2539	271	270	99	1677	739	44067	44005
47	927804	2855	308	295	100	966	464	48033	47182
48	922715	2981	323	322	101	524	263	50191	50497
49	918925	3484	379	353	102	265	155	58491	53948
50	926502	3733	403	386	103	133	62	46617	57532
51	947362	4038	426	424					
52	991831	4551	459	465					

Table 4 Derivation of central rates of mortality (m_x) for 2000-02, England
(b) Females

Age	Exposed to risk	Deaths 2000-02	Crude death rates per 100,000	Graduated death rates per 100,000	Age	Exposed to risk	Deaths 2000-02	Crude death rates per 100,000	Graduated death rates per 100,000
0	826216	4038	489	488	53	1065933	3522	330	339
1	839530	288	34	31	54	1038108	3867	373	372
2	857272	154	18	22	55	980794	3915	399	408
3	876846	148	17	16	56	898393	4002	445	447
4	885775	130	15	13	57	870857	4217	484	490
5	890922	106	12	12	58	823777	4265	518	537
6	896925	93	10	11	59	768401	4436	577	589
7	907944	87	10	10	60	737503	4808	652	646
8	929123	102	11	10	61	734265	5213	710	709
9	948086	88	9	10	62	742429	5730	772	780
10	957487	92	10	10	63	734371	6214	846	858
11	954234	97	10	10	64	720158	6768	940	946
12	948327	102	11	10	65	701981	7140	1017	1045
13	941067	91	10	11	66	681357	7846	1152	1155
14	931365	136	15	13	67	666202	8327	1250	1278
15	917650	133	14	16	68	660659	9314	1410	1417
16	903374	213	24	22	69	665044	10366	1559	1572
17	891908	226	25	27	70	665149	11604	1745	1747
18	873230	262	30	28	71	656147	12999	1981	1944
19	879087	260	30	28	72	637326	14119	2215	2167
20	902076	241	27	28	73	620713	15411	2483	2417
21	910249	266	29	29	74	608051	16999	2796	2701
22	893755	246	28	29	75	597606	18754	3138	3022
23	878629	266	30	29	76	584273	20484	3506	3385
24	882822	258	29	30	77	570175	22206	3895	3797
25	906742	289	32	31	78	563427	24108	4279	4265
26	942343	336	36	33	79	563379	26806	4758	4797
27	988975	338	34	34	80	556653	29826	5358	5402
28	1043216	360	35	36	81	489679	28988	5920	6091
29	1095605	408	37	39	82	404349	27071	6695	6859
30	1123988	438	39	42	83	338482	25061	7404	7709
31	1142036	549	48	45	84	317314	27348	8619	8649
32	1152581	566	49	49	85	314977	30364	9640	9685
33	1175595	619	53	53	86	298480	31853	10672	10824
34	1189954	722	61	58	87	272569	32507	11926	12074
35	1201768	789	66	63	88	237177	31708	13369	13444
36	1202037	827	69	68	89	201916	30273	14993	14940
37	1192310	836	70	75	90	171296	28107	16408	16572
38	1171631	975	83	82	91	141770	26028	18359	18347
39	1145181	996	87	90	92	114741	23542	20518	20274
40	1109397	1060	96	99	93	89858	20679	23013	22361
41	1076686	1137	106	109	94	68054	16982	24954	24617
42	1049607	1255	120	120	95	50081	13834	27623	27048
43	1026484	1422	139	132	96	35855	10809	30146	29664
44	999853	1440	144	145	97	25144	8141	32378	32471
45	969493	1560	161	160	98	17141	6147	35861	35443
46	953092	1752	184	176	99	11253	4299	38203	38548
47	943918	1847	196	193	100	7115	2949	41448	41776
48	939282	2008	214	212	101	4314	1928	44692	45112
49	934648	2221	238	233	102	2497	1205	48258	48540
50	944085	2518	267	256	103	1380	725	52536	52043
51	965292	2672	277	281					
52	1010187	3163	313	309					

Table 5 Derivation of central rates of mortality (m_x) for 2000-02, Wales

(a) Males

Age	Exposed to risk	Deaths 2000-02	Crude death rates per 100,000	Graduated death rates per 100,000	Age	Exposed to risk	Deaths 2000-02	Crude death rates per 100,000	Graduated death rates per 100,000
0	48030	267	556	563	53	64465	343	532	519
1	49730	26	52	42	54	62610	363	580	573
2	51090	12	23	27	55	59095	381	645	634
3	53037	6	11	19	56	54812	397	724	702
4	54264	9	17	14	57	53966	397	736	779
5	54990	6	11	11	58	52140	444	852	866
6	55530	9	16	9	59	49046	507	1034	965
7	56359	3	5	8	60	46945	524	1116	1078
8	58077	1	2	7	61	45844	567	1237	1205
9	59666	5	8	7	62	45659	616	1349	1349
10	60472	7	12	7	63	44817	683	1524	1509
11	60997	2	3	8	64	43850	766	1747	1685
12	60755	4	7	10	65	42802	791	1848	1880
13	60663	14	23	12	66	41303	847	2051	2094
14	59905	16	27	16	67	39937	922	2309	2331
15	59599	8	13	24	68	38450	928	2414	2591
16	58364	19	33	37	69	37903	1082	2855	2876
17	57067	38	67	54	70	37142	1188	3199	3190
18	54668	43	79	72	71	35972	1249	3472	3533
19	53697	52	97	85	72	34113	1306	3828	3909
20	56022	40	71	92	73	32737	1396	4264	4320
21	55651	60	108	93	74	31865	1556	4883	4768
22	51609	40	78	95	75	30907	1629	5271	5257
23	47192	46	97	96	76	29401	1749	5949	5789
24	45278	45	99	98	77	27264	1810	6639	6367
25	45755	51	111	100	78	25644	1782	6949	6995
26	46622	42	90	102	79	24266	1861	7669	7682
27	48379	58	120	104	80	22533	1919	8516	8435
28	51167	55	107	107	81	19193	1732	9024	9263
29	54194	66	122	109	82	15318	1504	9819	10172
30	55821	52	93	112	83	12329	1380	11193	11169
31	57104	53	93	115	84	10368	1382	13329	12265
32	57475	70	122	119	85	9380	1313	13998	13467
33	58864	93	158	123	86	8235	1225	14876	14788
34	60114	79	131	127	87	7031	1143	16257	16238
35	61794	94	152	133	88	5719	1064	18605	17830
36	62808	82	131	139	89	4530	928	20486	19577
37	62777	90	143	146	90	3705	802	21646	21466
38	62480	82	131	155	91	2814	593	21073	23474
39	61376	115	187	164	92	2083	501	24052	25603
40	60068	114	190	174	93	1491	411	27565	27852
41	58605	107	183	186	94	1029	308	29932	30219
42	57677	107	186	200	95	687	224	32606	32700
43	57008	126	221	216	96	445	153	34382	35293
44	55826	123	220	233	97	280	107	38214	37991
45	54713	125	228	253	98	168	70	41667	40787
46	54463	135	248	275	99	99	32	32323	43675
47	54832	166	303	299	100	57	23	40351	46644
48	55165	189	343	327	101	31	10	32258	49683
49	55357	202	365	357	102	16	11	68750	52782
50	56686	252	445	391	103	9	6	66667	55927
51	58604	227	387	429					
52	61687	287	465	471					

Table 5 Derivation of central rates of mortality (m_x) for 2000-02, Wales
(b) Females

Age	Exposed to risk	Deaths 2000-02	Crude death rates per 100,000	Graduated death rates per 100,000	Age	Exposed to risk	Deaths 2000-02	Crude death rates per 100,000	Graduated death rates per 100,000
0	45405	199	438	448	53	65418	201	307	354
1	47220	11	23	19	54	63637	266	418	390
2	48739	9	18	16	55	60309	264	438	431
3	50489	3	6	14	56	55885	262	469	475
4	51331	9	18	12	57	55095	316	574	525
5	52220	5	10	11	58	53053	288	543	580
6	52818	5	9	10	59	50577	315	623	640
7	53937	6	11	9	60	48247	312	647	707
8	55575	6	11	9	61	47266	384	812	782
9	56835	5	9	9	62	47033	349	742	864
10	57213	8	14	9	63	46747	429	918	956
11	56920	5	9	9	64	45889	491	1070	1058
12	57171	3	5	10	65	44930	515	1146	1171
13	57324	8	14	10	66	43863	540	1231	1296
14	56906	4	7	11	67	43193	667	1544	1435
15	56517	9	16	13	68	42758	624	1459	1590
16	55567	7	13	14	69	42568	740	1738	1762
17	55219	8	14	17	70	42397	779	1837	1953
18	54119	15	28	20	71	41882	944	2254	2167
19	55634	13	23	23	72	40817	1003	2457	2404
20	57444	18	31	26	73	40291	1102	2735	2668
21	55519	18	32	29	74	40101	1164	2903	2963
22	51067	23	45	31	75	39976	1389	3475	3292
23	46954	19	40	33	76	39156	1415	3614	3658
24	45426	10	22	35	77	37953	1533	4039	4067
25	45846	19	41	36	78	37323	1697	4547	4524
26	47443	11	23	38	79	37115	1880	5065	5034
27	50215	16	32	40	80	36568	2009	5494	5604
28	53655	14	26	42	81	32105	2036	6342	6241
29	56983	17	30	44	82	27137	1871	6895	6954
30	59071	43	73	47	83	22887	1846	8066	7750
31	60620	37	61	50	84	20982	1833	8736	8642
32	61466	26	42	53	85	20070	1990	9915	9640
33	62662	43	69	57	86	18574	2065	11118	10758
34	63386	36	57	61	87	16686	2082	12478	12011
35	64550	50	77	66	88	14484	2029	14009	13415
36	65398	49	75	71	89	12366	1901	15373	14989
37	65895	57	87	78	90	10584	1793	16941	16755
38	65441	55	84	85	91	8759	1690	19294	18737
39	64385	64	99	93	92	7090	1451	20465	20936
40	62566	75	120	102	93	5552	1345	24226	23292
41	61126	74	121	112	94	4205	1045	24851	25797
42	59913	71	119	123	95	3093	864	27934	28445
43	59239	92	155	135	96	2215	722	32596	31224
44	58035	90	155	149	97	1554	545	35071	34122
45	56469	85	151	164	98	1059	356	33617	37122
46	56073	125	223	180	99	695	250	35971	40206
47	56391	122	216	198	100	440	204	46364	43351
48	56875	131	230	218	101	266	100	37594	46534
49	57163	146	255	240	102	154	89	57792	49776
50	58161	135	232	264	103	85	42	49412	53083
51	59919	167	279	291					
52	62523	171	273	321					

2. Spline graduation of crude mortality rates for the English Life Table 16

by

Vladimir K. Kaishev*, Steven Haberman, Dimitrina S. Dimitrova,

Cass Business School, City University, London

Introduction

The graduation methods used for ELT 14 and ELT 15 were based on cubic spline fitting. In the ELT 14, a variable-knot cubic spline fitting method was applied. The number of knots and their locations at the age axis were assumed free parameters which were estimated by minimizing an appropriately defined c^2 criterion. The graduated cubic spline fits consisted of 12 (11) cubic polynomial pieces, for females (males), smoothly joint at some optimally selected ages (knot positions).

In ELT 15, a classical cubic spline smoothing method was applied, where each data observation was taken as a knot and a smoothness parameter, determining the degree o smoothness of the fit, was estimated. This approach avoids the necessity of estimating the number and location of the knots, but requires the estimation of the smoothness parameters and also some weight parameters attached to each data observation.

In graduating the ELT 16, we have decided to follow the variable-knot spline regression approach, and use a weighted least squares version of the GeD spline regression method, which has recently been proposed by Kaishev et al. (2006 a, b). The method determines the degree of the spline fit and also the number and positions of its knots, according to some optimality criterion. As it will be demonstrated in the last section, it has produced quadratic spline fits of the logs of the crude mortality data, which are not overly parameterized and can be evaluated, for any arbitrary age, using a calculator. In what follows, we state the variable-knot spline regression problem, give a brief summary of the GeD spline method and illustrate its use in estimating mortality curves, based on a set of crude central mortality rates.

The method

Suppose that $\{x_1, ..., x_N\}$ are age points in the range $[a, b]$, satisfying $a < x_1 < ... < x_N < b$ at which the crude mortality data have been collected and that the transformed crude mortality rates are

$$y_i = \log\left(\overset{\circ}{m}_{x_i}\right), \quad i = 1, 2, ..., N. \tag{1}$$

It is assumed that there is an (unknown) functional relationship between the (response) variable y and the age variable x of the form

$$y = f(x) + \epsilon,$$

where ϵ is a random (observation) error variable with zero mean and some variance σ^2 and $f(\cdot)$ is an unknown function, approximated with a n-th order (degree n - 1) polynomial spline. Let us recall that a spline function $f(\boldsymbol{t}_{k,n}; x)$ on an interval [a, b], consists of pieces of polynomials of a certain degree, n - 1, and these pieces are smoothly joint together at some points

$$\boldsymbol{t}_{k,n} = \{t_1 = \ ... = t_n = a < t_{n+1} < ... < t_{n+k} < t_{n+k+1} = b = \ ... = t_{2\,n+k}\},$$

called the knots of the spline. The spline function admits the representation

$$f(\boldsymbol{t}_{k,n}; x) = \boldsymbol{\theta}' \boldsymbol{N}_n(x) = \sum_{i=1}^{p} \theta_i N_{i,n}(x),$$

where $\boldsymbol{\theta} = (\theta_1, \ ..., \theta_p)'$ is a vector of (unknown) regression coefficients and $\boldsymbol{N}_n(x) = (N_{1,n}(x), \ ..., N_{p,n}(x))'$, $p =$ n + k, are certain basis (spline) functions, known as B-splines of order n. B-splines are splines defined on $t_{k,n}$ through the Mansfield-De Boor-Cox recurrence relation

$$N_{i,1}(t) = \begin{cases} 1 & \text{if} \quad t_i \le t < t_{i+1} \\ 0 & \text{otherwise} \end{cases},$$

$$N_{i,n}(t) = \frac{t - t_i}{t_{i+n-1} - t_i} N_{i,n-1}(t) + \frac{t_{i+n} - t}{t_{i+n} - t_{i+1}} N_{i+1,n-1}(t)$$

from which it can be seen that $N_{i,n}(t) = 0$ for $t \notin [t_i, t_{i+n}]$. In order to emphasize the dependence of the spline regression $f(\boldsymbol{t}_{k,n}; x)$ on $\boldsymbol{\theta}$, we will further use the notation $f(\boldsymbol{t}_{k,n}, \boldsymbol{\theta}; x)$.

In the current context, the spline regression estimation problem can be formulated as follows. Based on a sample of observations of the crude mortality rates, $\{y_i\}_{i=1}^{N}$, at the age points $\{x\}_{i=1}^{N}$, estimate the degree n - 1 of the spline, the number of knots, k, the set of knots, $\boldsymbol{t}_{k,n}$, and the regression coefficients $\boldsymbol{\theta}$, so that the estimated spline curve of the crude mortality rates is sufficiently smooth but at the same time captures all the peculiarities of the shape of the functional relationship in (1). In addition, it is required that the curve is not overly parametrized, i.e., it does not include too many knots and regression coefficients $\boldsymbol{\theta}$. To solve this estimation problem, we have used the method of geometrically designed variable-knot regression splines, called GeD splines or simply GeDS, which was recently developed by Kaishev et al. (2006 a,b).

Since often, as in the case of crude mortality data, observations, y_i, i = 1, …, N, may have different variances, σ_i^2, i.e., some observations may be more accurate than others,

a weighted version of the least squares estimation of the regression coefficients $\boldsymbol{\theta}$ is applied. The latter copes well with this, so called heteroscedasticity of the observation errors ϵ_i, $i = 1, \ldots,$ N . Under the weighted least squares spline regression, given a fixed position of the knots, $t_{k,n}$ the sum of the weighted squared residuals

$$S = \sum_{i=1}^{N} w_{ii} \left(y_i - f\left(\boldsymbol{t}_{k,n}, \boldsymbol{\theta}; x_i\right)\right)^2$$

is minimized with respect to the parameters, $\boldsymbol{\theta}$, where each weight, w_{ii}, is inversely proportional to the error variance, i.e. $w_{ii} = \frac{1}{\sigma_i^2}$. In practice, the variances, σ_i^2 are not known and the weights are often estimated as $w_{ii} = \frac{1}{r_i^2}$, where r_i are the residuals obtained by fitting a non-weighted least squares spline regression in the first instance. This weighted least squares fitting approach is known as feasible generalized least squares, see White (1980). In order to graduate the crude mortality data for the ELT 16 life table, we have applied the feasible generalized least squares in conjunction with the GeD variable-knot spline regression. Thus, we have been able to estimate the location of the knots, $\boldsymbol{t}_{k,n}$ and given a fixed set, $\boldsymbol{t}_{k,n}$, to produce feasible generalized least squares estimates of the parameters, $\boldsymbol{\theta}$, as

$$\hat{\boldsymbol{\theta}} = \left(\boldsymbol{F}^{\boldsymbol{T}}\, \boldsymbol{W}\, \boldsymbol{F}\right)^{-1} \boldsymbol{F}^{\boldsymbol{T}}\, \boldsymbol{W}\, \boldsymbol{Y},$$

where $\boldsymbol{F}$ is an $N \times p$ matrix obtained by evaluating the vector of B-spline functions, $\boldsymbol{N}_n(x) = (N_{1,n}(x), \ldots, N_{p,n}(x))'$ at the N, data values, x_i, $i = 1, \ldots, N$, and where $Y = (y_1, \ldots, y_N)'$ and $\boldsymbol{W}$ is an $N \times N$ diagonal matrix of weights with diagonal elements, w_{ii}, $i = 1, \ldots, N$.

The GeDS method is motivated by ideas from the field of Computer Aided Geometric Design and exploits the geometric relationship between the spline curve and its so called control polygon. The latter closely follows the shape of the spline curve and is formed by a set of control points, connected with straight lines, and the x and y coordinates of the control points are directly expressed through the knots $\boldsymbol{t}_{k,n}$ of the spline and its coefficients $\boldsymbol{\theta}$. Thus, estimation of $\boldsymbol{t}_{k,n}$ and $\boldsymbol{\theta}$ is interpreted as adjustment of the position of the control points, hence the position of the curve, so as to match the shape of the data.

The GeD spline estimation method includes two stages. In stage A, a linear least squares spline fit to the data is constructed, and viewed as the (initial) position of the control polygon of a higher order (n > 2) smooth spline curve. In stage B, the optimal set of knots of this higher order spline curve is found, so that its control polygon is as close to the initial polygon of stage A as possible and finally, the least squares estimates of the regression coefficients of this curve are found. An adaptive knot location scheme for generating linear spline fits has been developed in stage A such that at each step of stage

A, a knot is placed where a certain bias dominated measure is maximal. This stage is equipped with a novel stopping rule which serves as a model selector. The optimal knots, $\tilde{\boldsymbol{t}}_{\tilde{k},n}$, defined in stage B ensure that the higher order spline curve is nearly a variation diminishing (i.e., shape preserving) spline approximation to the linear fit of stage A. Error bounds for this approximation are derived in Kaishev et al. (2006 b).

The GeDS method produces simultaneously linear, quadratic, cubic (and possibly higher order) spline fits with one and the same number of B-spline regression functions. Hence, the order $\tilde{n}$ is chosen so that the spline fit $\hat{f}\left(\tilde{\boldsymbol{t}}_{\tilde{k},\tilde{n}}, \hat{\boldsymbol{\theta}}; x\right)$ has the minimum residual sum of squares. In this way, along with the number of knots and their locations, the degree of the spline is also estimated. This is an important feature of the GeD estimation method which is rarely offered by other spline estimation procedures. For a complete description of the method, its statistical properties and the optimality of the estimated knot locations, we refer the interested reader to Kaishev et al. (2006 a, b).

Fitting the crude mortality rates

Crude mortality data have been collected for ages up to 108 for males and 112 for females. In order to extrapolate the spline fits beyond these maximum ages and close the life tables at a limiting age $\omega = 121$, the spline fits were constrained at age 120 so that $m_{120} = 2$. The latter is obtained by setting $q_{120} = 1$ and using the approximation

$$q_x \approx \frac{m_x}{1 + 0.5\, m_x}, \qquad (2)$$

which holds for $x = 120$ if l_x is assumed linear in the age interval [120, 121].

Since the observed mortality rate is quite high in the first year of life and sharply drops in the next year, this would create a severe constraint on the fits at the first year of age. In order to avoid this difficulty, the first observation y_0 for age $x_0 = 0$ has been excluded from the data sample, as has also been done in graduating the ELT 15.

All graduations in the English Life Tables 16 have been performed using the weighted version of the GeDS method as described in the previous section. We present here results of the spline fitting over the age interval $[a, b] = [1, 120]$ for England and Wales, males and females. A summary of the resulting quadratic $(\tilde{n} - 1 = 2)$ spline fits with number of knots, $\tilde{k} = 18$ for both males and females, and sets of knot values

$\tilde{\boldsymbol{t}}_{18,3}$ = {1, 1, 1, 3.83, 5.41, 8.18, 12.51, 16.36, 19.77, 28.41, 38.60, 45.31, 51.47, 56.70, 62.54, 70.23, 77.54, 83.55, 89.57, 94.55, 98.13, 120, 120, 120}

and

$\tilde{\boldsymbol{t}}_{18,3}$ = {1, 1, 1, 4.89, 8.94, 12.23, 14.00, 15.86, 17.69, 21.88, 28.34, 35.34, 44.26, 55.08, 64.94, 71.52, 77.35, 82.85, 87.19, 91.82, 95.83, 120, 120, 120}

respectively, is presented in Tables 1 and 2. For any fixed age x, such that

$t_{3+i} \le x \le t_{3+i+1}$, i =1,…,18, the predicted value, y, of the log of the crude mortality rate is given by the following quadratic equation,

$$y = c_{i,0} + c_{i\,1}\, x + c_{i\,2}\, x^2,$$

where $c_{i\,j}$, j = 0, 1, 2 are the estimated least squares regression coefficients, in this piecewise-quadratic polynomial representation of the final quadratic spline fit, $\hat{f}\left(\tilde{\boldsymbol{t}}_{18,3}, \hat{\boldsymbol{\theta}}; x\right)$. Given the estimated set of knots, $\tilde{\boldsymbol{t}}_{18,3}$, and parameters, $\hat{\boldsymbol{\theta}}$, these coefficients are easily obtained. For the fits for England and Wales, males and females, their values, for all the 19 quadratic polynomial pieces, are given in Tables 1 and 2 respectively. Note that the polynomial piece for the first year of age is linear and is not part of the quadratic spline fit, produced by the GeDS method. Let us note also, that the first three and the last three knots in the set, $\tilde{\boldsymbol{t}}_{18,3}$, are auxiliary knots, and their choice does not affect the fitting process and the final spline fits.

Construction of the life table

The life table has been constructed using the graduated (and extrapolated) mortality rates m_x, x = 1, …, 120, in order to calculate the values of q_x. For the purpose, the following methodology has been applied. The value of qx has been derived directly from the data on births and infant deaths.
To obtain q1, as in ELT 15, we have applied the formula

$$q_1 \approx m_1 \left[\frac{1 + (1/2)m_2}{1 + (1/12)(7m_1 + 5m_2) + (1/3)m_1 m_2} \right].$$

In order to convert the graduated central mortality rates m_x into q_x values, for ages x = 2, …, 100, we have used the approximation

$$q_x = m_x \left[\frac{1 - (1/2)m_{x-1}}{1 + (5/12)(m_x - m_{x-1}) - (1/6)m_x m_{x-1}} \right],$$

proposed by McCutcheon (1975-77) which holds under the assumption that l_x is quadratic over the age range $[x-1,\ x+1]$. Beyond age 100, i.e. for ages $100 < x \le 120$, we have adopted the approximation formula (2), since, as mentioned in the previous section, in extrapolating m_x up to age x = 120, by setting q_{120} = 1 and m_{120} = 2, we have implied that the approximation (2) holds. Furthermore, as shown by McCutcheon (1975-77), the approximation formula (2) is exact when l_x is a linear function of age over the interval $x,\ x+1$. As known, the assumption of linearity of l_x is a consequence of the assumption of uniform distribution of deaths over the age range, $[x,\ x+1]$, which does not contradict with the observed survival data, for ages, $100 < x \le 120$.

Finally, let us note that m_x rates for any, not necessarily integer age value x, can be calculated from the piece-wise quadratic polynomial representation of the $\log_{10}$ of the crude mortality rate spline fits and used to obtain other mortality functions which might be of interest.

Figure 1 (a) shows the graduated curves of the central rates of mortality m_x , superimposed on the actual crude rates for each age. Figure 1(b) shows a magnified portion of the curves for ages 1 to 30. Figure 2 shows the graph of the values of q_x derived from the graduated values of m_x using the above formula.

Figure 1(a) ELT16 – Graph of $\log_{10}$ (m_x)

Figure 1(b) ELT16 – Graph of $\log_{10}$ (m_x) for young ages

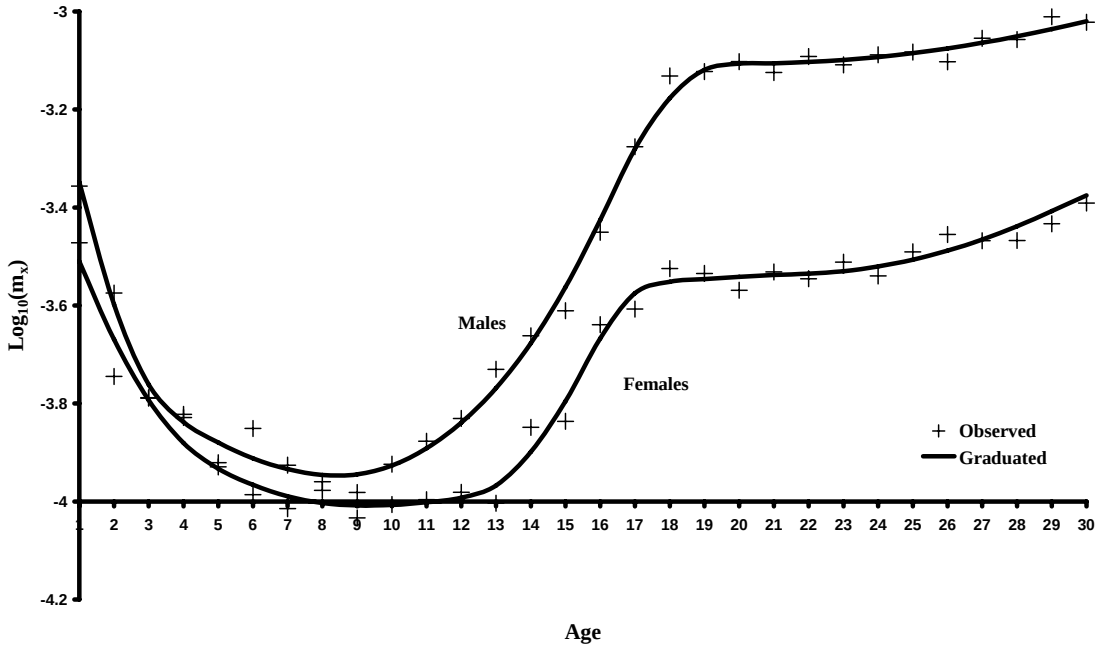
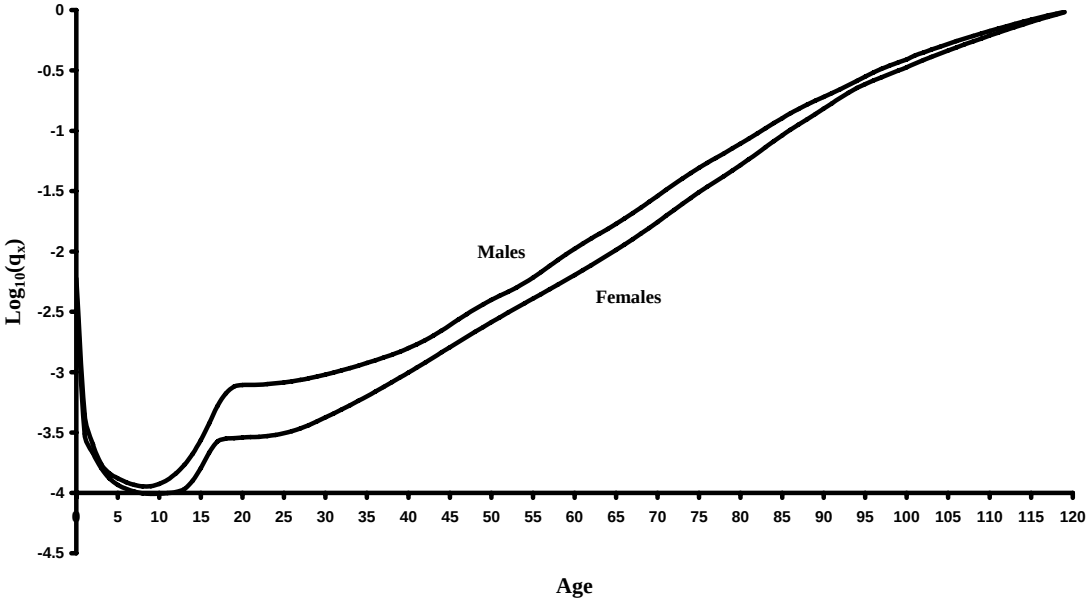


Figure 2 ELT16 – Graph of $\log_{10}(q_x)$



3. References

Kaishev, V. K., Dimitrova, D. S., Haberman, S. and Verrall R. (2006 a). Geometricallydesigned, variable knot regression splines: Asymptotics and inference. Statistical Res.Paper 28, Cass Business School, City University, London.

Kaishev, V. K., Dimitrova, D. S., Haberman, S. and Verrall R. (2006 b). Geometricallydesigned, variable knot regression splines: Variation diminishing optimality of knots.Statistical Res. Paper 29, Cass Business School, City University, London.

McCutcheon, J. J. (1975-1977). Some elementary Life Table approximations. Transac tions of the Faculty of Actuaries 35 , 297-301

Thatcher, A.R., Kannisto, V. & Andreev, K (2002) The survivor ratio method for for estimating number at high ages. *Demography.* 6.

White, H. (1980). A Heteroskedasticity-Consistent Covariance Matrix Estimator and aDirect Test for Heteroskedasticity, Econometrica 48 (4), 817-838.

4. Appendix 1 GeD Spline fit: knots and coefficients

England & Wales

Coefficients-Males					Coefficients-Females				
Knot range		c_0	c_1	c_2	Knot range		c_0	c_1	c_2
0.00	1.00	-2.2195	-1.1292		0.00	1.00	-2.3133	-1.1969	0.0000
1.00	3.83	-3.0124	-0.3797	0.0434	1.00	4.89	-3.3154	-0.2126	0.0178
3.83	5.41	-3.5816	-0.0821	0.0045	4.89	8.94	-3.6306	-0.0837	0.0046
5.41	8.18	-3.5601	-0.0900	0.0052	8.94	12.23	-3.8460	-0.0355	0.0019
8.18	12.51	-3.3364	-0.1447	0.0086	12.23	14.00	-0.8134	-0.5316	0.0222
12.51	16.36	-2.9200	-0.2113	0.0112	14.00	15.86	-2.6753	-0.2656	0.0127
16.36	19.77	-12.0710	0.9077	-0.0230	15.86	17.69	-14.9647	1.2838	-0.0361
19.77	28.41	-2.7443	-0.0360	0.0009	17.69	21.88	-3.8088	0.0226	-0.0005
28.41	38.60	-3.0500	-0.0145	0.0005	21.88	28.34	-2.5301	-0.0943	0.0022
38.60	45.31	-1.3737	-0.1013	0.0016	28.34	35.34	-3.8882	0.0016	0.0005
45.31	51.47	-7.3121	0.1608	-0.0013	35.34	44.26	-4.1894	0.0186	0.0003
51.47	56.70	0.8767	-0.1574	0.0018	44.26	55.08	-5.2083	0.0647	-0.0002
56.70	62.54	-8.2208	0.1635	-0.0010	55.08	64.94	-3.6351	0.0075	0.0003
62.54	70.23	-1.5805	-0.0489	0.0007	64.94	71.52	-2.0067	-0.0426	0.0007
70.23	77.54	-8.6381	0.1521	-0.0007	71.52	77.35	-8.8507	0.1488	-0.0007
77.54	83.55	-1.0629	-0.0433	0.0005	77.35	82.85	0.2963	-0.0877	0.0009
83.55	89.57	-12.1876	0.2230	-0.0011	82.85	87.19	-11.7095	0.2021	-0.0009
89.57	94.55	3.4633	-0.1265	0.0009	87.19	91.82	-1.2055	-0.0389	0.0005
94.55	98.13	-17.4660	0.3163	-0.0014	91.82	95.83	-23.6596	0.4502	-0.0022
98.13	120.00	-4.2895	0.0477	-0.0001	95.83	120.00	-3.0098	0.0193	0.0001

Wales

Coefficients-Males					Coefficients-Females				
Knot range		c_0	c_1	c_2	Knot range		c_0	c_1	c_2
0.00	1.00	-2.2550	-1.1129	0.0000	0.00	1.00	-2.3583	-1.3637	0.0000
1.00	7.24	-3.1538	-0.2285	0.0144	1.00	17.13	-3.6352	-0.0917	0.0050
7.24	14.55	-3.2848	-0.1923	0.0119	17.13	24.12	-6.5948	0.2537	-0.0051
14.55	20.16	-8.3565	0.5047	-0.0120	24.12	37.28	-2.6613	-0.0724	0.0016
20.16	26.30	-3.9595	0.0686	-0.0012	37.28	49.16	-6.0635	0.1101	-0.0008
26.30	34.84	-2.7157	-0.0260	0.0006	49.16	53.41	0.0572	-0.1389	0.0017
34.84	43.98	-2.1343	-0.0594	0.0011	53.41	67.07	-4.9190	0.0475	0.0000
43.98	62.96	-3.6230	0.0083	0.0003	67.07	82.79	-3.9514	0.0186	0.0002
62.96	78.91	-5.9519	0.0823	-0.0003	82.79	90.67	-8.6766	0.1328	-0.0005
78.91	83.99	-2.6986	-0.0002	0.0003	90.67	100.62	-6.0611	0.0751	-0.0002
83.99	91.56	-8.6417	0.1413	-0.0006	100.62	120.00	-7.0939	0.0956	-0.0003
91.56	120.00	-4.2732	0.0459	-0.0001					

Scotland

Knot range		Coefficients-Males c_0	c_1	c_2	Knot range		Coefficients-Females c_0	c_1	c_2
0.00	1.00	-2.2083	-1.2763	0.0000	0.00	1.00	-2.3156	-1.1647	0.0000
1.00	8.94	-3.3699	-0.1221	0.0075	1.00	8.76	-3.3098	-0.1811	0.0106
8.94	12.42	-3.6250	-0.0650	0.0043	8.76	12.21	-3.2072	-0.2045	0.0119
12.42	16.17	-1.0835	-0.4745	0.0208	12.21	15.78	-4.3589	-0.0158	0.0042
16.17	20.51	-12.5257	0.9410	-0.0230	15.78	18.60	-10.8367	0.8052	-0.0218
20.51	25.45	-2.1193	-0.0737	0.0018	18.60	22.38	-2.4978	-0.0917	0.0023
25.45	31.69	-3.7299	0.0528	-0.0007	22.38	28.28	-3.2664	-0.0230	0.0008
31.69	41.52	-1.6716	-0.0771	0.0013	28.28	39.01	-2.8865	-0.0498	0.0012
41.52	53.89	-2.9794	-0.0141	0.0006	39.01	50.15	-5.3036	0.0741	-0.0004
53.89	63.62	-5.8574	0.0927	-0.0004	50.15	61.89	-4.1205	0.0269	0.0001
63.62	73.62	-3.1909	0.0089	0.0002	61.89	71.82	-3.8446	0.0180	0.0002
73.62	82.79	-6.3785	0.0955	-0.0004	71.82	80.65	-4.4335	0.0344	0.0001
82.79	94.61	-3.6492	0.0296	0.0000	80.65	91.64	-6.5749	0.0875	-0.0003
94.61	120.00	-6.0516	0.0804	-0.0002	91.64	120.00	-6.2018	0.0794	-0.0002

Northern Ireland

Knot range		Coefficients-Males c_0	c_1	c_2	Knot range		Coefficients-Females c_0	c_1	c_2
0.00	1.00	-2.2425	-1.1993		0.00	1.00	-2.3222	-1.2995	0.0000
1.00	7.53	-3.2982	-0.1540	0.0104	1.00	6.67	-3.4926	-0.1378	0.0086
7.53	13.45	-3.2561	-0.1652	0.0111	6.67	15.72	-3.6233	-0.0986	0.0056
13.45	19.10	-7.1104	0.4081	-0.0102	15.72	22.67	-6.5055	0.2681	-0.0060
19.10	27.72	-3.9463	0.0768	-0.0015	22.67	29.27	-2.0290	-0.1268	0.0027
27.72	40.43	-1.3894	-0.1077	0.0018	29.27	40.58	-3.4153	-0.0321	0.0011
40.43	61.88	-4.1074	0.0268	0.0002	40.58	50.68	-6.9287	0.1411	-0.0011
61.88	85.73	-5.0777	0.0582	-0.0001	50.68	65.23	-2.8541	-0.0197	0.0005
85.73	96.94	-3.9703	0.0323	0.0000	65.23	82.66	-4.8026	0.0400	0.0001
96.94	120.00	-9.7016	0.1506	-0.0006	82.66	120.00	-7.3207	0.1010	-0.0003

England

Knot range		Coefficients-Males			Knot range		Coefficients-Females		
		c_0	c_1	c_2			c_0	c_1	c_2
0.00	1.00	-2.2176	-1.1389		0.00	1.00	-2.3109	-1.1881	0.0000
1.00	4.35	-3.0537	-0.3376	0.0349	1.00	4.73	-3.2873	-0.2324	0.0207
4.35	7.90	-3.6446	-0.0662	0.0037	4.73	8.68	-3.6666	-0.0720	0.0038
7.90	12.39	-3.3779	-0.1337	0.0080	8.68	13.02	-3.5666	-0.0950	0.0051
12.39	16.35	-2.8035	-0.2264	0.0117	13.02	15.80	-1.1097	-0.4724	0.0196
16.35	19.84	-12.0726	0.9076	-0.0230	15.80	17.59	-15.9780	1.4095	-0.0400
19.84	28.11	-2.5873	-0.0485	0.0011	17.59	20.75	-3.5658	-0.0015	0.0001
28.11	38.18	-3.1197	-0.0106	0.0005	20.75	25.80	-3.1709	-0.0396	0.0010
38.18	45.06	-1.3399	-0.1039	0.0017	25.80	30.18	-1.9484	-0.1344	0.0029
45.06	51.20	-7.3951	0.1649	-0.0013	30.18	35.36	-4.9221	0.0627	-0.0004
51.20	56.53	0.6775	-0.1504	0.0018	35.36	44.19	-3.7889	-0.0014	0.0005
56.53	62.40	-7.9196	0.1537	-0.0009	44.19	54.20	-5.5566	0.0786	-0.0004
62.40	70.11	-1.9202	-0.0386	0.0006	54.20	64.04	-3.5990	0.0064	0.0003
70.11	76.49	-7.3961	0.1176	-0.0005	64.04	71.58	-2.3534	-0.0325	0.0006
76.49	80.14	-7.8976	0.1307	-0.0006	71.58	77.46	-8.2017	0.1309	-0.0006
80.14	84.27	1.5808	-0.1058	0.0009	77.46	82.97	-1.2734	-0.0480	0.0006
84.27	90.00	-12.3030	0.2237	-0.0010	82.97	88.02	-7.0004	0.0901	-0.0002
90.00	95.40	-1.1102	-0.0250	0.0003	88.02	94.29	-11.2238	0.1860	-0.0008
95.40	102.44	-11.5142	0.1931	-0.0008	94.29	101.96	-8.3379	0.1248	-0.0005
102.44	120.00	-0.6262	-0.0195	0.0002	101.96	120.00	-2.6595	0.0134	0.0001